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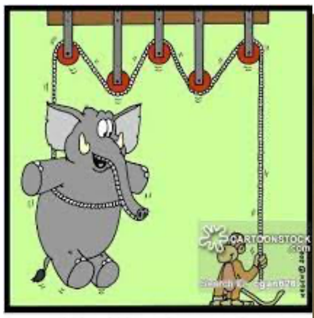
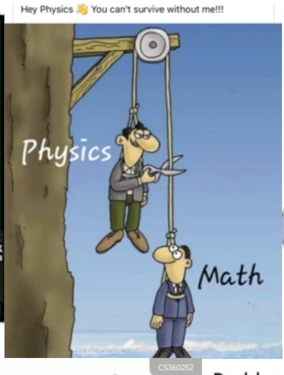
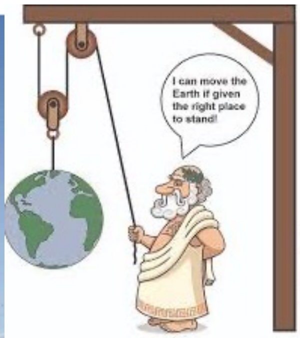
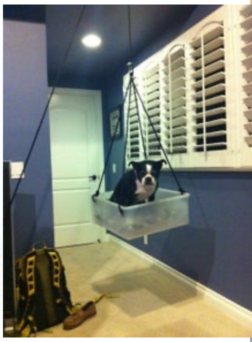
Pulleys – Vertical & Horizontal

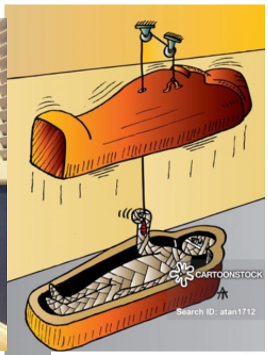


Alright, alright, you've won your bet: You can lift me with one hand...



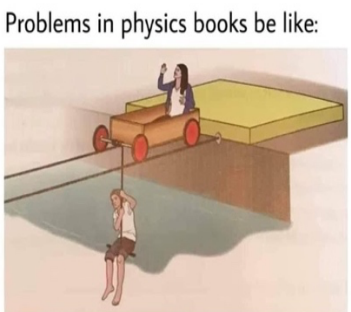


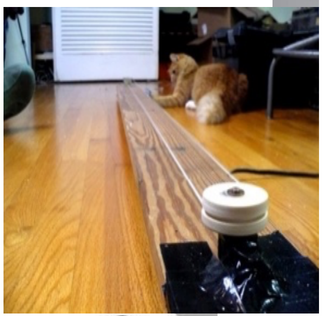


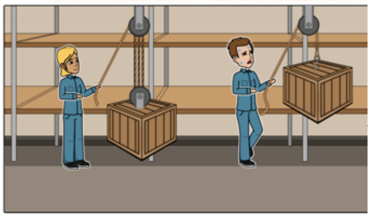


"It's the cheapest multi-story carpark in town."

"The rule is that you should never eat more than you can lift, and you're not allowed to use a pulley."







"Do we look like we're going up?"

Table of Contents

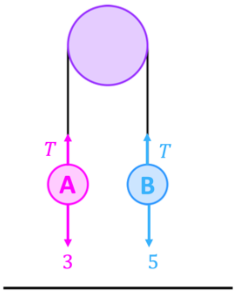
This is a long worksheet to cater for students that want extra practice. If you want a shortcut, but still be sure to cover one of each type then follow the pink highlighted questions. If you are already good at these you can start straight at the Gold section.

1 Bronze

1.1 Vertical – Known Masses

1) **Given diagram**

Particles *A* and *B*, of masses 3 kg and 5 kg are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. The system is at rest, the string taut and its straight parts vertical. Find The acceleration of the particles and the tension in the string before *B* reaches the floor.

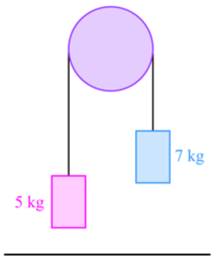


2) **Draw your own diagram**

Particles *A* and *B*, of masses 3 kg and 2 kg respectively, are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. The system is at rest, the string taut and its straight parts vertical. Find The acceleration of the particles and the tension in the string before *A* reaches the floor.

1.1.1 With SUVAT –Distance Travelled and Greatest Height

3) The following diagram shows two masses of 5 kg and 7 kg respectively connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest and the 7 kg mass reaches the ground after 3 seconds s.



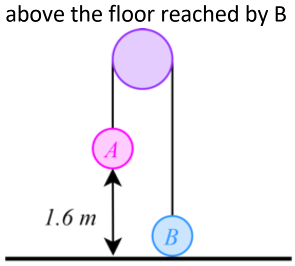
Calculate

i. The acceleration of the masses while the string remains taut

ii. Find the tension in the string

iii. The total distance moved by the 5 kg mass **before it comes to instantaneous rest**, assuming that it does not reach the pulley

4) Particles *A* and *B*, of masses 0.35 kg and 0.15 kg respectively, are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. The system is at rest with *B* held on the horizontal floor, the string taut and its straight parts vertical. *A* is at a height of 1.6 m above the floor (see diagram). *B* is released and the system begins to move; *B* does not reach the pulley. Find
   i. The acceleration of the particles and the tension in the string before *A* reaches the floor
   ii. The **greatest height** above the floor reached by B

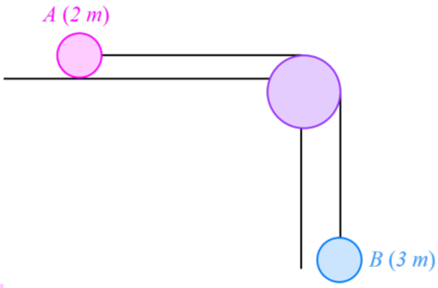


5) Two particles, A and B, of mass 15 kg and 12 kg respectively, are attached to the ends of a light inextensible string which passes over a fixed, smooth pulley. The particles are held at rest so that the string is taut and they are both 6 m above the horizontal ground
   i. Find the acceleration of the particles immediately after they are released from rest and the tension in the string

   Particle A strikes the ground without rebounding. Particle B moves freely under gravity without striking the pulley. Find ii.
   ii. the **time between** particle A hitting the ground and particle B coming to rest for the first time
   iii. Find the **distance that B travelled between** particle A hitting the ground and particle B coming to rest for the first time
   iv. State how you have used in modelling that the string is light

1.2 Horizontal - Known Masses

6) Two particles A and B have masses 2m and 3m respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a smooth horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Particle B hangs at rest vertically below the pulley with the string taut, as shown above. Particle A is released from rest. Assuming that A has not reached the pulley, find

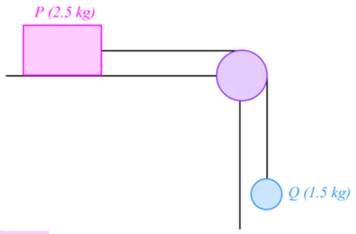


   i. The acceleration of B
   ii. The tension in the string in terms of m
   iii. The **magnitude and direction of the force exerted on the pulley** by the string

7) **Draw your own diagram**
   Two particles A and B off mass 5 kg and 3 kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table and the string passes over a small smooth pulley which is fixed at the edge of a table. Particle B hangs freely. The friction between A and the table is 24.5 N. The system is released from rest. Find
   i. The acceleration of the system
   ii. The tension of the string
   iii. The **magnitude of the force exerted on the pulley** by the string

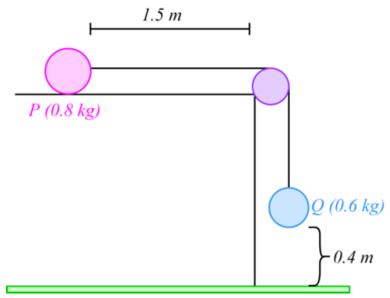
1.2.1 With SUVAT – Finding the Acceleration / Time Taken To Hit The Pulley

8) **Find the acceleration first using SUVAT**
A box P of mass 2.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a sphere Q of mass 1.5 kg which hangs freely below the pulley. The magnitude of the frictional force between P and the table is k N. The system is released from rest with the string taut. After release, Q descends a distance of 0.8 in 0.75 s. Modelling P and Q as particles



i. Calculate the acceleration of Q
Hint: use SUVAT to get a, resolving won't give you a since too many unknowns
ii. Show that the tension in the string is 10.4 N
iii. Find the value of k
iv. Show how in your calculations you have used the information that the string is inextensible

9) A small ball, P of mass 0.8 kg, is held at rest on a smooth horizontal table and it attached to one end of a thin rope. The rope passes over a pulley that is fixes at the edge of the table. The other end of the rope is attached. The other end of the is attached to another small ball Q of mass 0.6kg that hangs freely below the pulley. Ball P is released from rest with the rope taut with P at a distance of 1.5m from the pulley and with Q at a height of 0.4m above the ground. Ball Q hits the floor and doesn't rebound.
The balls are modelled as particles the rope as a light inextensible string and the pulley as smooth and small.

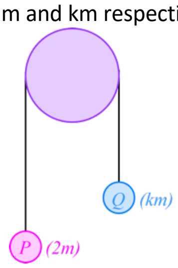


Using this model,
i. Show that the acceleration of Q as it falls is 4.2 ms^{-2}
ii. Find the **time taken by P to hit the pulley** from the instant when P is released
iii. State one limitation of the model that will affect the accuracy of your answer to part i.

2 Silver

2.1 Vertical – Unknown Masses

10) Two small balls, P and Q, have masses 2m and km respectively, where $k < 2$.



The balls are attached to the ends of a string that passes over a fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown. The system is released from rest and, in the subsequent motion, P moves downwards with an acceleration of magnitude $\frac{5g}{7}$. The balls are modelled as particles moving freely. The string is modelled as being light and inextensible. The pulley is modelled as being small and smooth.

Using the model,

i. find, in terms of m and g, the tension in the string

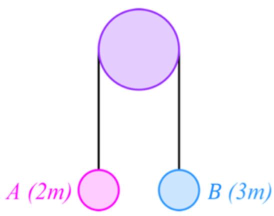
ii. explain why the acceleration of Q also has magnitude $\frac{5g}{7}$

iii. find the value of k

iv. Identify one limitation of the model that will affect the accuracy of your answer to part iii.

2.1.1 With SUVAT – Greatest Height and Taut Again

11) Two particles *A* and *B* have masses 2*m* and 3*m* respectively. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and *A* and *B* are above a horizontal plane, as shown above. The system is released from rest.



i. Show that the tension in the string immediately after the particles are released is $\frac{12}{5}mg$

After descending 1.5 m, *B* strikes the plane and is immediately brought to rest. In the subsequent motion, *A* does not reach the pulley.

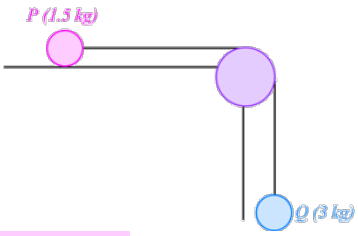
ii. Calculate the **greatest height** of A above the ground

iii. Find the time and distance travelled by *A* between the instant when *B* strikes the plane and the instant when the string next **becomes taut**.

2.2 Horizontal

2.2.1 With Friction (year 2 only)

12) **Year 2 (coefficient of friction):** Two particles *P* and *Q* have masses 1.5 kg and 3 kg respectively. The particles are attached to the ends of a light inextensible string. Particle *P* is held at rest on a fixed rough horizontal table. The coefficient of friction between *P* and the table is $\frac{1}{5}$. The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle *Q* hangs freely at rest vertically below the pulley, as shown. Particle *P* is released from rest with the string taut and slides along the table.



Assuming that *P* has not reached the pulley, find

i. the tension in the string during the motion

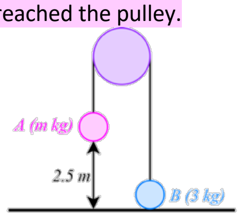
ii. the **magnitude and direction of the resultant force exerted on the pulley** by the string

3 Gold

3.1 Vertical

3.1.1 Using SUVAT Multiple Times

13) **Finding acceleration first using SUVAT**
Two particles A and B have masses m kg and 3 kg respectively where $m > 3$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initally A is 2.5 m above the ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in the diagram. After A has been descending for 1.25 seconds, it strikes the ground. Particle A reaches the ground before has reached the pulley.

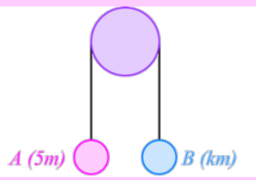


i. Show that the acceleration of B as it ascends is 3.2 ms^{-2}
ii. Find the tension in the string as A descends
iii. Show that $m = \frac{65}{11}$
iv. State how you have used the information that the string is inextensible

When A strikes the ground it does not rebound and the string becomes slack. Particle B then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

v. Calculate the **greatest height** of B above the ground
vi. Find the time between the instant when A strikes the ground and the instant when the string **becomes taut** again
vii. Find the **magnitude and direction of the resultant force exerted on the pulley**

14) Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown above. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

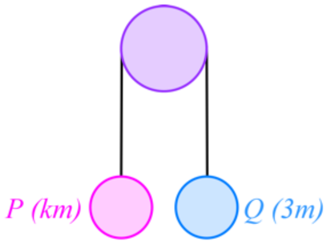


i. Show that the tension in the string as A descends is $\frac{15}{4}mg$
ii. Find the value of k
iii. State how you have used the information that the pulley is smooth

After descending for 1.2s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley

iv. Find the **greatest height** reached by B above the plane
v. Find the time between the instant when A strikes the ground and the instant when the string **becomes taut** again

15) Two particles P and Q have masses km and $3m$ respectively where $k < 3$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with P and Q at the same height above a horizontal plane, as shown in the diagram. The system is released from rest. After release, Q descends with acceleration $\frac{1}{3}g$



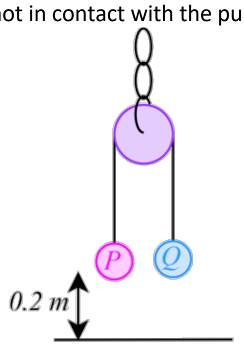
i. Calculate the tension in the string as Q descends
ii. Show that $k = 1.5$
iii. State how you have used the information that the pulley is smooth

After descending for 1.8 s, the particle Q reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between P and the pulley is such that, in the subsequent motion, P does not reach the pulley.

iv. Show that the **greatest height**, in metres, reached by P above the plane is 1.26g

3.1.2 More Than 1 Tension (suspended from a chain)

16) **Finding acceleration first using SUVAT**
A small smooth pulley is suspended from a fixed point by a light chain. A light inextensible string passes over the pulley. Particles *P* and *Q*, of masses 0.3 kg and *m* kg respectively, are attached to the opposite ends of the string. The particles are released from rest at a height of 0.2 m above horizontal ground with the string taut; the portions of the string not in contact with the pulley are vertical.



P strikes the ground with speed 1.4 ms^{-1}. Subsequently *P* remains on the ground, and *Q* does not reach the pulley.

i. Calculate the acceleration of *P* while it is in motion and the corresponding tension in the string.
ii. Find the value of m

It is given that the mass of the pulley is 0.5 kg. State the magnitude of the tension in the chain which supports the pulley when

a) *P* is in motion.
b) *P* is at rest on the ground and *Q* is moving upwards.

iii. Calculate the **greatest height** of *Q* above the ground

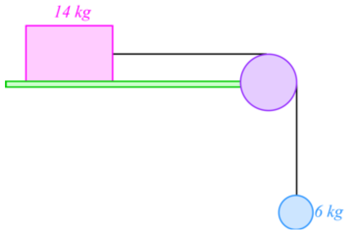
3.1.3 With SUVAT - Speed hits the pulley and total distance travelled

17) A box *A* of mass 0.8 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a smooth pulley fixed at the edge of the table. The other end of the string is attached to a sphere *B* of mass 1.2 kg, which hangs freely below the pulley. The magnitude of the frictional force between *A* and the table is *F* N. The system is released from rest with the string taut. After release, B descends a distance of 0.9 m in 0.8 s.Modelling A and B as particles, calculate
    i. the acceleration of B
    ii. the tension in the string
    iii. the value of F

    Sphere B is 0.9 m above the ground when the system is released. Given that A does not reach the pulley and the frictional force remains constant throughout,
    iv. find the **total distance** travelled by A

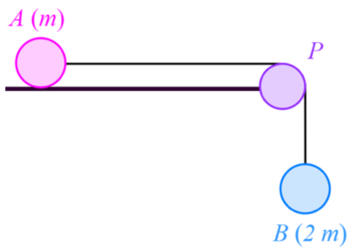
3.1.4 With Friction (year 2 only)

18) **Year 2 (coefficient of friction)**: A block, of mass 14 kg, is held at rest on a rough horizontal surface. The coefficent of friction between the block and the surface is 0.25. A light intensible string, which passes over a fixed smooth peg, is attached to the block. The other end of the string is attached to a particle, of mass 6 kg, which is hanging at rest. The block is released and begins to accelerate.



    i. Find the magnitude of the fricton force acting on the block
    ii. By forming two equations of motion, one for the block and one for the particle, show that the magnitude of the acceleration of the block and the particle is 1.225 ms^{-2}.
    iii. Find the tension in the string
    iv. When the block is released, it is 0.8 metres from the peg. Find the **speed of the block when it hits the peg**.
    v. When the block reaches the peg, the string breaks and the particle falls a further 0.5 metres to the ground. Find the **speed of the particle when it hits the ground**

19) **Year 2 (coefficient of friction)**: Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough horizontal table. The string passes over a small smooth pulley P fixed on the edge of the table. The particle B hangs freely below the pulley, as shown. The coefficient of friction between A and the table is μ. The particles are released from rest with the string taut. Immediately after release, the magnitude of the acceleration of A and B is $\frac{4}{9}$g. By writing down separate equations of motion for A and B.



    i. Find the tension in the string immediately after the particles begin to move
    ii. show that $\mu=\frac{2}{3}$

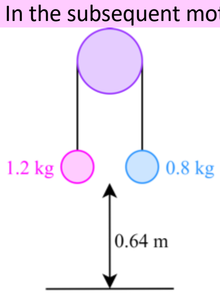
    When *B* has fallen a distance *h*, it hits the ground and does not rebound. Particle *A* is then a distance $\frac{1}{3}h$ from the P.
    iii. Find the **speed** of *A* as it reaches *P*
    iv. State how you have used the information that the string is light

4 Diamond

4.1 Vertical

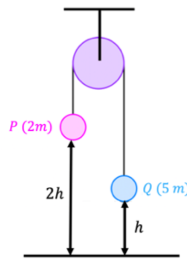
4.1.1 SUVAT Many Times (Hardest SUVAT Type)

20) Two particles of masses 1.2 kg and 0.8 kg are connected by a light inextensible string that passes over a fixed smooth pulley. The particles hang vertically. The system is released from rest with both particles 0.64 m above the floor (see diagram). In the subsequent motion the 0.8 kg particle does not reach the pulley.



i. Show that the acceleration of the particles is 1.96 ms^{-2} and find the tension in the string
ii. Find the **total distance** travelled by the 0.8 kg particle during the first second after the particles are released

21) A ball P of mass $2m$ is attached to one end of a string. The other end of the string is attached to a ball Q of mass $5m$. The string passes over a fixed pulley. The system is held at rest with the balls hanging freely above and the string taut. The hanging parts of the string are vertical with P at a height of $2h$ above the horizontal ground and with Q at a height of h above the ground, as shown. The system is released from rest.



In the subsequent motion, Q does not rebound when it hits the ground and P does not hit the pulley.
The balls are modelled as particles
The string is modelled as being light and inextensible
The pulley is modelled as being small and smooth
Air resistance is modelled as being negligible
Using this model,

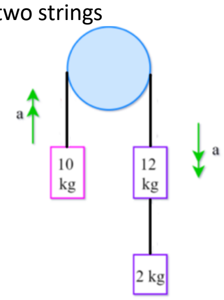
a. Write down an equation of motion for P
b. Write down an equation of motion for Q
c. Find, in term of h only, the height of above the ground at which P first comes to instantaneous rest
d. State one limitation of modelling the balls as particles that could affect your answer to part ii.

In reality, the string will not be inextensible

e. State how this would affect the acceleration of the particles

4.1.2 More Than 1 Tension (2 objects)

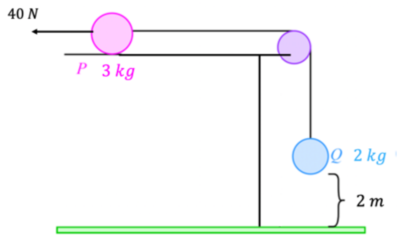
22) Three objects connected by two strings, one of which is passed round a pulley. Find the acceleration of the system and the tensions in the two strings



4.2 Horizontal

4.2.1 SUVAT Many Times (Hardest SUVAT Type)

23) Two small balls *P* and *Q* have masses 3 kg and 2 kg respectively. The balls are attached to the ends of a string. *P* is held at rest on a rough horizontal surface. The string passes over a pulley which is fixed at the edge of the surface. *Q* hangs vertically below the pulley at a height of 2 m above a horizontal floor.



The system is initially at rest with the string taut. A horizontal force of magnitude 40 N acts on *P* as shown in the diagram.
P is released and moves directly away from the pulley. A constant frictional force of magnitude 8 N opposes the motion of *P*. It is given that *P* does not leave the horizontal surface and that *Q* does not reach the pulley in the subsequent motion.
The balls are modelled as particles, the pulley is modelled as being small and smooth, and the string is modelled as being light and inextensible.

i. Show that the magnitude of the acceleration of each particle is 2.48 m s
ii. Find the tension in the string.

When the balls have been in motion for 0.5 seconds, the string breaks.

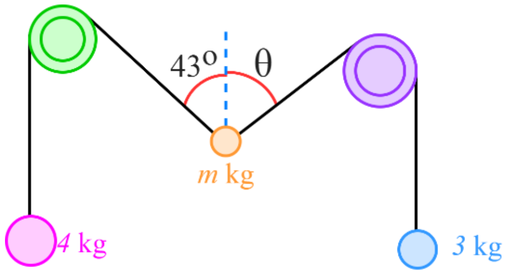
iii. Find the additional time that elapses until *Q* hits the floor.
iv. Find the speed of *Q* as it hits the floor.
v. Write down the magnitude of the normal reaction force acting on *Q* when *Q* has come to rest on the floor
vi. State one improvement that could be made to the model.

4.2.2 Harder Algebra

24) Two particles, A of mass of masses m_1 kg and B of mass m_2 kg are connected by a light inextensible string. The string passes over a smooth pulley, P. A sits on a rough horizontal table, where the coefficient of friction between A and the table is μ, and B lies directly below P. Given that $m_2 > \mu m_1$, show that the acceleration of the system is $\frac{g(m_2-\mu m_1)}{m_1+m_2}$

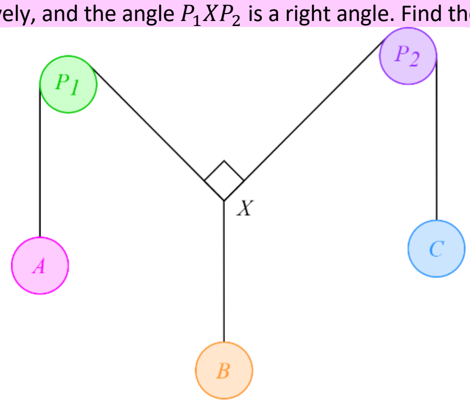
4.3 Vertical - 2 Pulleys and Diagonal Forces (year 2 only)

25) Two light, inextensible strings are attached to a particle of mass m kg. Each string passes over a fixed, smooth, light pulley. The other end of one string is attached to a particle of mass 4 kg. The other end of the second string is attached to a particle of mass 3 kg. The diagram shows the system in its equilibrium position. The angles marked on the diagram are between the strings and the vertical

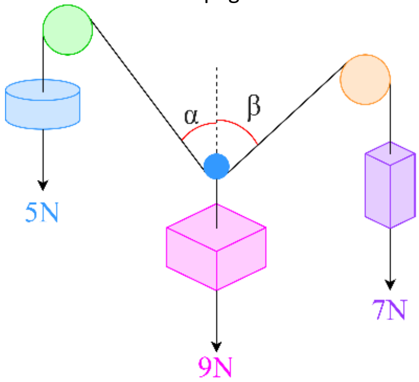


i. Calculate the tension in each of the strings
ii. Show that $\theta = 65.4°$, correct to three significant figures
iii. Find m

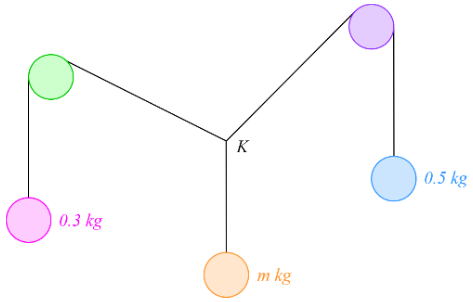
26) The diagram shows three particles A, B and C hanging freely in equilibrium, each being attached to the end of a string. The other ends of the three strings are tied together and are at the point X. The strings carrying A and C pass over smooth fixed horizontal pegs p_1 and p_2 respectively. The weights of A, B and C are 5.5 N, 7.3 N and W N respectively, and the angle P_1XP_2 is a right angle. Find the angle AP_1X and the value of W.



27) Three strings are knotted together at one end, and parcels of weights 5N, 7N, and 9 N are attached to the other ends. The first two strings are placed over smooth horizontal pegs, and the third parcel hangs freely, as shown. The system is in equilibrium. Find the angle which the first 2 strings make with the vertical between the knot and the pegs



28) The diagram shows three strings, which are tied in a knot K. Two of the strings pass over smooth pulleys and have particles of mass 0.3kg and 0.5 kg attached to them at the ends opposite to K. The other string has a particle of mass m kg attached to it at the end opposite to K. The system is at rest



i. In the case m=0.7, find the angle made by the sloping part of each string with the upward vertical
ii. Give a reason why m<0.8
iii. In the case where m=0.4, show that part of one of the strings is horizontal
iv. If the pulleys were at the same horizontal level, give a reason why m>0.4